TABLE 1

l, m	m, kg	R, m	v_0, m/sec	P_1	P_2	P_3	P_4	P_5	T_1	T_2	T_3	T_4	T_5
				kN					$c\cdot10^{-3}$				
0,9	20	0,065	3,65	335	399	222	356	324	0,354	0,310	0,518	0,322	0,348
0,6	13,1	0,063	1,73	120	128	80	134	121	0,492	0,310	0,459	0,265	0,298
0,6	13,1	0,063	3,84	286	331	188	298	273	0,274	0,260	0,427	0,265	0,287
0,6	13,1	0,063	5,20	397	474	260	404	372	0,240	0,249	0,417	0,265	0,280
0,3	4,86	0,063	4,20	193	207	123	199	182	0,197	0,175	0,270	0,161	0,175
0,3	4,86	0,063	5,56	252	289	266	263	243	0,165	0,165	0,260	0,161	0,170
0,3	4,86	0,063	6,50	300	349	196	308	286	0,149	0,160	0,258	0,161	0,170

experiments reported in [4], and the results of our calculations were compared with the experimental results. Steel rods of length ℓ and mass m fell with initial speed v_0 onto a base of duraluminum D1-T. The ends of the rods were curved (with radius of curvature R). The basic impact characteristics are shown in Table 1. Here P_1, P_2, P_3, P_4, and P_5 are the maximum values of the contact force as determined, experimentally in [4], by the Sears model in [1], by Kil'chevskii's elastic-plastic model in [3], and by the rigid-plastic and elastic-plastic local deformation models, respectively; T_1, T_2, T_3, T_4, and T_5 are the times of impact. Comparing results, we see that the Sears theory, based on an elastic model due to Hertz, gives a value for P_{max} greater, on the average by 20-30%, in comparison with the experimental value, and it gives a lower value for T. The theory, based on Kil'chevskii's elastic-plastic model [3], yields a value for P_{max} lower by 30-40% and a larger value for T. The theory proposed here yields results which differ from the experimental results by 2-6%. In the case of the rigid-plastic model, a particular case of the elastic-plastic model, the basic impact parameters are given in the explicit form (12) and results of calculations differ from experiment by 2-12%.

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COMPACT REPRESENTATION OF THE FUNDAMENTAL SOLUTION OF THE INTERNAL LAMB PROBLEM ON A FREE SURFACE

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The now-classical expression derived [1] for the Green's function of the problem of ground surface displacements induced by an explosion in a homogeneous containment medium is written in the form of a three-term sum. One of the terms is the Boussinesq solution for a half-space, which assumes that the disturbance propagates instantaneously [2]; another term contains typical Rayleigh components, and the third represents certain real integrals. This representation of the Green's function is convenient from the standpoint of the physical treatment of the propagation of seismic waves in a medium and affords a rapid and efficient means of calculating the displacements far from the wave front. However, it is nonoptimal for calculating the displacements near the wave front at a large distance from the detonation epicenter, where the indicated terms strongly suppress one another in the vicinity of the front.

In the present article we describe an attempt to surmount this difficulty by deriving a more compact representation of the Green's function in question without incurring such a

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mutual cancellation of the individual terms. This is accomplished by changing the previously adopted [1] order of integration with respect to the frequency and the wave vector in the basic equations, essentially in the spirit of Lamb's method. The feasibility of switching the order of integration in this way with the proper choice of contours of integration has been proved [3] as part of a comparison of the method of partial separation of variables with the Smirnov–Sobolev method of complex solutions [4]. In this sense the present study is a natural elaboration of the approach suggested by Petrashen' et al. [3], and it provides a concrete illustration of the well-known equivalence of different forms of the solution of seismological problems [5].

1. In carrying out the intermediate calculations, it is more convenient to work with the Green's function $\chi_{r,z}$ corresponding to a delta-function displacement signal, and then to convert later to the customary linearly increasing displacement signal or a unit-step signal in the form of a Heaviside function for the velocities. We therefore consider the functions

$$\chi_r(\tau)=\frac{R_0^2c_p}{2\pi\gamma^2}\int\limits_{-\infty}^{\infty}dk_p e^{-iR_f k_p}k_p^2\int\limits_{C(k)}k^2J_1(kr)\frac{\nu_s e^{-\nu_p H}}{\left(k^2-\frac{1}{2}k_s^2\right)^2-k^2\nu_p\nu_s}dk,$$
$$\chi_z(\tau)=\frac{R_0^2c_p}{2\pi\gamma^2}\int\limits_{-\infty}^{\infty}dk_p e^{-iR_f k_p}k_p^2\int\limits_{C(k)}kJ_0(kr)\frac{\left(k^2-\frac{1}{2}k_s^2\right)e^{-\nu_p H}}{\left(k^2-\frac{1}{2}k_s^2\right)^2-k^2\nu_p\nu_s}dk. \tag{1.1}$$

Here H is the charge-placement depth, r is the epicentral distance from an observation point on the ground surface to the detonation center, R_0 is the elastic radius at which the source function is specified, $R_f = c_p\tau + R_0$ is the radius of the wave front, the time τ is measured from the instant at which the source begins to take effect on a sphere of the indicated radius R_0, $\gamma = c_s/c_p$ is the ratio of the propagation velocities of transverse (shear) and longitudinal (pressure) waves in the containment medium, $k_{p,s} = \omega/c_{p,s}$, ω is the angular frequency, and k is the horizontal component of the wave vector.

The quantities $\nu_{p,s} = \sqrt{k^2 - k_{p,s}^2}$ characterize the attenuation of inhomogeneous or complex seismic waves with distance from the source (from the ground surface in the case of Rayleigh waves) along the vertical axis. In the cases $k^2 < k_p^2$ for longitudinal waves and $k^2 < k_s^2$ for transverse waves these quantities go over to the vertical components $\varkappa_{p,s}$ of the wave vectors, multiplied by an imaginary unit. The contour of integration C(k) in Eq. (1.1) is chosen so that $\varkappa_{p,s}=-\sqrt{k_{p,s}^2-k^2}$ for positive frequencies and $\varkappa_{p,s}=\sqrt{k_{p,s}^2-k^2}$ for negative frequencies. This choice of contour corresponds to the condition that only waves traveling away from the free surface are present at large depths in the medium. The upward direction is considered the z axis. The origin is located at the epicenter of the explosion.

These conditions are satisfied if the contour C(k) is drawn parallel to the real axis for positive frequencies – below it for positive frequencies and above it for negative frequencies. These contours of integration correspond to the cuts proposed by Aki and Richards [6] in the complex plane of the variable k for the solution of problems of this kind. On the real axis these cuts are situated on the intervals $0 < k < |k_p|$ for longitudinal waves and $0 < k < |k_s|$ for transverse waves.

The Green's functions (1.1) are adopted as basic so that the domain of zero frequencies, where the contour of integration changes, will not contribute to them. We also note that the selected contours, as postulated, make the functions $\chi_{r,z}$ real-valued. Consequently, we can in fact limit the integration in Eq. (1.1) to positive frequencies only and then double the real part of the result.

For all positive frequencies the contour of integration with respect to k is unique. For this reason, setting $k = k_p y$, we can rewrite Eq. (1.1) in the form

$$\chi_r(\tau)=\frac{R_0^2c_p}{\pi\gamma^2}\operatorname{Re}\left\{\int\limits_{C(y)}\frac{y^2\sqrt{y^2-a^2}\,dy}{\left(y^2-\frac{1}{2}a^2\right)^2-y^2\sqrt{y^2-1}}\int\limits_0^{\infty}J_1(ryk_p)e^{-\left(H\sqrt{y^2-1}+iR_f\right)k_p}k_p^2\,dk_p\right\},$$
$$\chi_z(\tau)=\frac{R_0^2c_p}{\pi\gamma^2}\operatorname{Re}\left\{\int\limits_{C(y)}\frac{y\left(y^2-\frac{1}{2}a^2\right)dy}{\left(y^2-\frac{1}{2}a^2\right)^2-y^2\sqrt{y^2-1}\sqrt{y^2-a^2}}\int\limits_0^{\infty}J_0(ryk_p)e^{-\left(H\sqrt{y^2-1}+iR_f\right)k_p}k_p^2\,dk_p\right\}. \tag{1.2}$$

The cuts for the square-root functions in this expression are situated on the real axis on the intervals (0, 1) and (0, a), where $a = 1/\gamma$. The contour of integration C(y) runs parallel to and below the real axis.

We now replace one of the factors k_p in Eq. (1.2) by the time derivative, replace the other one by space derivatives with respect to r for χ_r and with respect to H for χ_z, and then compute the inner integrals. As a result, we have

$$\chi_r(\tau) = -\frac{R_0^2}{\pi\gamma^2}\frac{\partial^2}{\partial\tau\,\partial r}\operatorname{Re}\left\{\int\limits_{C(y)}\frac{iy\sqrt{y^2-a^2}\,dy}{\left(y^2-\frac{1}{2}a^2\right)^2-y^2\sqrt{y^2-1}\sqrt{y^2-a^2}}\frac{1}{R(y)}\right\},$$
$$\chi_z(\tau) = -\frac{R_0^2}{\pi\gamma^2}\frac{\partial^2}{\partial\tau\,\partial H}\operatorname{Re}\left\{\int\limits_{C(y)}\frac{iy\left(y^2-\frac{1}{2}a^2\right)dy}{\left(y^2-\frac{1}{2}a^2\right)^2-y^2\sqrt{y^2-1}\sqrt{y^2-a^2}}\frac{1}{\sqrt{y^2-1}\,R(y)}\right\}, \tag{1.3}$$

where

$$R(y) = \sqrt{r^2y^2+(H\sqrt{y^2-1}+iR_f)^2}.$$

For the ensuing discussion it is useful to augment the contour C(y) with a quarter-circle of infinitely large radius connecting the positive real semiaxis to the negative imaginary semiaxis. The integrals along this arc in Eq. (1.3) are finite, but do not depend on the time. The functions $\chi_{r,z}$ are therefore left unchanged by this modification of the contour of integration.

We also note that if the wave has not yet reached the observation point, i.e., if the hypocentral distance $R = \sqrt{r^2 + H^2} > R_f$, the branch points of the root R(y) are situated on the real axis. In fact transforming this root to the form

$$R(y) = R\sqrt{\frac{r^2}{R^2}\left(1-\frac{R_f^2}{R^2}\right)-\left(\sqrt{1-y^2}-\frac{R_fH}{R^2}\right)^2}, \tag{1.4}$$

we find that the expression under the square-root sign vanishes at the points y_0 determined by the equation

$$\sqrt{1-y_0^2} = \frac{R_fH}{R^2} \pm \frac{r}{R}\sqrt{1-\frac{R_f^2}{R^2}}. \tag{1.5}$$

It is readily verified that the right-hand side of this equation is smaller than unity. Consequently, the branch points of the given root are located on the interval of the real axis (−1, 1) and do not prohibit the contour C(y) from being arbitrarily deformed in the fourth quadrant. We make this contour coincide with the negative imaginary semiaxis. On it, by virtue of the remark concerning the right-hand side of Eq. (1.5), the expression under the square-root sign in Eq. (1.4) is negative. Consequently, under the condition $R_f < R$ the integrals in Eq. (1.3) are purely imaginary and do not contribute to the functions $\chi_{r,z}$, which are therefore equal to zero for $R_f < R$ in accordance with the causality principle.

If $R_f > R$, the integrands in Eq. (1.3) are purely imaginary on the interval of the real axis $0 < y < 1$, because now $R^2(y) < 0$ by virtue of Eq. (1.4). This fact can be utilized to shift the origin of the contour of integration to the point $y = 1$ and to set $y = \sqrt{1 + x^2}$. As a result, the functions $\chi_{r,z}$ are written in the form

$$\chi_r(\tau) = -\frac{R_0^2}{\pi\gamma^2}\frac{\partial^2}{\partial\tau\,\partial r}\operatorname{Re}\left\{\int\limits_{C(x)}\frac{ix\sqrt{1-a^2+x^2}\,dx}{(1-\frac{1}{2}a^2+x^2)^2-x(1+x^2)\sqrt{1-a^2+x^2}}\frac{\theta(R_f-R)}{R(x)}\right\},$$
$$\chi_z(\tau) = -\frac{R_0^2}{\pi\gamma^2}\frac{\partial^2}{\partial\tau\,\partial H}\operatorname{Re}\left\{\int\limits_{C(x)}\frac{i\left(1-\frac{1}{2}a^2+x^2\right)dx}{\left(1-\frac{1}{2}a^2+x^2\right)^2-x(1+x^2)\sqrt{1-a^2+x^2}}\frac{\theta(R_f-R)}{R(x)}\right\}, \tag{1.6}$$

where

$$R(x) = \sqrt{r^2(1+x^2)+(Hx+iR_f)^2} = R\sqrt{\left(x+i\frac{R_fH}{R^2}\right)^2-\frac{r^2}{R^2}\left(\frac{R_f^2}{R^2}-1\right)}. \tag{1.7}$$

The contour $C(x)$ is analogous to the contour $C(y)$ augmented, as explained above, by a quarter-circle of infinite radius in the fourth quadrant. The unit function $\theta(R_f - R)$ in Eq. (1.6) emphasizes the fact that the given estimates of the functions $\chi_{r,z}$ are valid for $R_f \geqslant R$. Otherwise $\chi_{r,z} \equiv 0$.

It follows from Eq. (1.7) that the expression under the square-root sign vanishes at the points $x_{1,2} = -i\frac{R_f H}{R^2} \pm \frac{r}{R}\sqrt{\frac{R_f^2}{R^2} - 1}$ and on the negative imaginary axis, so that the integrals along it do not contribute to $\chi_{r,z}$. We therefore have two contours of integration, which are equivalent to the contour $C(x)$ and are suitable for calculating these functions. One of them corresponds to the classical trinomial representation of the Green's function. It consists of the upper and lower edges of the cut of the function $\sqrt{1 - a^2 + x^2}$, a circle of infinitely small radius about the Rayleigh pole determined by the zero of the denominator of the integrands on the real axis, and a semicircle of infinitely large radius connecting the positive and negative imaginary semiaxes.

A more compact monomial representation of the Green's function is obtained if the contour $C(x)$ coincides with the negative imaginary semiaxis. In this case the significant (from the point of view functions $\chi_{r,z}$) components of the integrals (1.6) are determined by the integrals along the upper and lower edges of the projection of the cut of $R(x)$ into the fourth quadrant. Here, introducing the new variable of integration ϑ, which is defined as

$$ix = \zeta = \frac{R_f H}{R^2} + i\frac{r}{R}\sqrt{\frac{R_f^2}{R^2} - 1}\sin\vartheta, \tag{1.8}$$

we obtain

$$\begin{gathered}\chi_r(\tau) = \frac{2R_0^2}{\pi\gamma^2}\frac{\partial^2}{\partial\tau\,\partial r}\operatorname{Re}\left\{\frac{\theta(R_f - R)}{R}\int_0^{\pi/2}\frac{\zeta\sqrt{\zeta^2 + a^2 - 1}\,d\vartheta}{\left(\zeta^2 + \frac{1}{2}a^2 - 1\right)^2 + \zeta(1-\zeta^2)\sqrt{\zeta^2 + a^2 - 1}}\right\},\\ \chi_z(\tau) = \frac{2R_0^2}{\pi\gamma^2}\frac{\partial^2}{\partial\tau\,\partial H}\operatorname{Re}\left\{\frac{\theta(R_f - R)}{R}\int_0^{\pi/2}\frac{\left(\zeta^2 + \frac{1}{2}a^2 - 1\right)d\vartheta}{\left(\zeta^2 + \frac{1}{2}a^2 - 1\right)^2 + \zeta(1-\zeta^2)\sqrt{\zeta^2 + a^2 - 1}}\right\}.\end{gathered} \tag{1.9}$$

2. The presence of the space derivatives in Eqs. (1.9) makes it difficult to use these equations in calculations. However, the derivatives are easily eliminated. In this regard, we note that the integrands in Eqs. (1.9), which we denote by $K_{r,z}(\zeta)$, depend only on ζ. According to Eq. (1.8),

$$\begin{gathered}\frac{\partial\zeta}{\partial r} = -\frac{H}{R^2}\left(\frac{R_f r}{R^2} - i\frac{H}{R}\sqrt{\frac{R_f^2}{R^2} - 1}\sin\vartheta\right) - \frac{R_f r}{R^2}\frac{\partial\zeta}{\partial R_f},\\ \frac{\partial\zeta}{\partial H} = \frac{R_f}{R^2} - \frac{H}{R^2}\zeta - \frac{R_f H}{R^2}\frac{\partial\zeta}{\partial R_f},\quad \frac{\partial\zeta}{\partial R_f} = \frac{H}{R^2} + \frac{iR_f r\sin\vartheta}{R^2\sqrt{R_f^2 - R^2}}.\end{gathered}$$

Consequently,

$$\begin{gathered}\frac{\partial K_r(\zeta)}{\partial r} = \frac{r}{R^2}K_r(\zeta) - \frac{\partial}{\partial R_f}\left[K_r(\zeta)\left(\frac{R_f r}{R^2} - i\frac{H}{R}\sqrt{\frac{R_f^2}{R^2} - 1}\sin\vartheta\right)\right] + \frac{iR_f H}{R^2\sqrt{R_f^2 - R^2}}\frac{\partial K_r(\zeta)\cos\vartheta}{\partial\vartheta},\\ \frac{\partial K_z(\zeta)}{\partial H} = \frac{H}{R^2}K_z(\zeta) - \frac{\partial K_z(\zeta)\,\zeta}{\partial R_f} - \frac{iR_f r}{R^2\sqrt{R_f^2 - R^2}}\frac{\partial K_z(\zeta)\cos\vartheta}{\partial\vartheta}.\end{gathered}$$

Rewriting Eqs. (1.9) with this fact in mind, we find that

$$\chi_r(\tau) = -\frac{R_0}{c_p}\frac{\partial^2 G_r(\tau)}{\partial\tau^2},\quad \chi_z(\tau) = -\frac{R_0}{c_p}\frac{\partial^2 G_z(\tau)}{\partial\tau^2}.$$

The functions $G_{r,z}(\tau)$ themselves have the significance of the Green's functions corresponding to a solitary signal in the form of a Heaviside unit step function for the velocities. They are equal to zero for $R_f < R$, and for $R_f \geqslant R$ we have

$$G_r(\tau) = \frac{2R_0}{\pi\gamma^2 R}\operatorname{Re}\int_0^{\pi/2}\frac{\zeta\sqrt{\zeta^2 + a^2 - 1}\left[\frac{R_f r}{R^2} - i\frac{H}{R}\sqrt{\frac{R_f^2}{R^2} - 1}\sin\vartheta\right]d\vartheta}{\left(\zeta^2 + \frac{1}{2}a^2 - 1\right)^2 + \zeta(1-\zeta^2)\sqrt{\zeta^2 + a^2 - 1}},$$

$$G_z(\tau) = \frac{2R_0}{\pi\gamma^2 R}\,\mathrm{Re}\int_0^{\pi/2} \frac{\zeta\left(\zeta^2+\frac{1}{2}a^2-1\right)d\vartheta}{\left(\zeta^2+\frac{1}{2}a^2-1\right)^2+\zeta(1-\zeta^2)\sqrt{\zeta^2+a^2-1}}. \tag{2.1}$$

At the wave front ($R_f = R$) these functions suddenly change to

$$\begin{aligned} G_r(\tau_f) &= \frac{R_0}{\gamma^2 R}\,\frac{\frac{Hr}{R^2}\sqrt{a^2-\frac{r^2}{R^2}}}{\left(\frac{1}{2}a^2-\frac{r^2}{R^2}\right)^2+\frac{Hr^2}{R^3}\sqrt{a^2-\frac{r^2}{R^2}}}, \\ G_z(\tau_f) &= \frac{R_0}{\gamma^2 R}\,\frac{\frac{H}{R}\left(\frac{1}{2}a^2-\frac{r^2}{R^2}\right)}{\left(\frac{1}{2}a^2-\frac{r^2}{R^2}\right)^2+\frac{Hr^2}{R^3}\sqrt{a^2-\frac{r^2}{R^2}}}. \end{aligned} \tag{2.2}$$

Equations (2.1) also provide a simple means for obtaining asymptotic estimates of the Green's functions in question at large times ($R_f \gg R$):

$$G_r^{ac}(\tau) = \frac{2R_0}{1-\gamma^2}\,\frac{R_f r}{R^3},\quad G_z^{ac}(\tau) = \frac{2R_0}{1-\gamma^2}\,\frac{R_f H}{R^3}. \tag{2.3}$$

These estimates represent the Boussinesq solution for a half-space and occur as one of the terms in the trinomial representation of the Green's function. For the horizontal component of the displacement they are on the order of R/H times greater than the value of the Green's function at the wave front according to Eq. (2.2). At large distances from the detonation center this condition requires increased accuracy of computation of the integral term in the trinomial. As mentioned in the introduction, this fact has been the stimulus for seeking a more compact representation of the fundamental solution of the Lamb problem with an internal source.

3. We have compared the Greens functions calculated according to Eqs. (2.1) with tabulated [2] values of these functions. In [2] the fundamental solutions, which we denote by $\hat{G}_{r,z}(\hat{\tau})$, are given in dimensionless scale units of length and time. They are related to the above-defined Green's functions by the equations

$$\hat{G}_{r,z}(\hat{\tau}) = \frac{1000}{30}\,\frac{1}{2}\,\gamma G_{r,z}(\tau),\quad \hat{\tau} = \frac{30}{1000}\,\frac{R_f}{R_0}.$$

A calculation according to these equations shows that agreement occurs in all significant figures except in isolated cases, which are unquestionably misprints. This test confirms not only the validity of the representation (2.1) for the Green's function in question, but also its correct computer software implementation.

4. As an example of the application of these equations, we give the displacements and mass velocities calculated by means of them on the free surface for an underground explosion at a depth H = 0.5 km in a medium with c_p = 5.5 km/sec and $\gamma = 1/\sqrt{3}$. The explosion is simulated by a Haskell source [7], which is specified on a sphere of radius R_0 = 0.1 km. This source is characterized by the reduced potential, which has the time dependence

$$\Phi_0(t) = \Phi_0(\infty)[1 - e^{-x}f(x)],$$

where $f(x) = 1 + x + x^2/2 + x^3/6 - B_H x^4$, where $x = \Omega_H t$, and $\Phi_0(\infty)$, Ω_H, and B_H are constants. Seeking to compare our own results with those published in [8], we let $\Omega_H = 1.45\ \mathrm{sec}^{-1}$ and $B_H = 0.5$. The constant $\Phi_0(\infty)$ is inconsequential, since the displacements in Figs. 1 and 2, like those in Figs. 5.13 and 5.14 in [8], are given in units of $\Phi_0(\infty)/H^2$. Accordingly, the mass velocities $u_{r,z}(t) = \dot{w}_{r,z}(t)$ (the dot signifies the time derivative) are given in Figs. 3 and 4 in units of $\Phi_0(\infty)/H^2 c$, where time is measured from the instant at which the signal reaches the observation point.

The displacements of the free surface are expressed in terms of the Green's functions (2.1) as follows:

$$w_{r,z}(t) = \int_0^{t-(R-R_0)/c_p} G_{r,z}(t-\tau)\,f_2(\tau)\,d\tau. \tag{4.1}$$

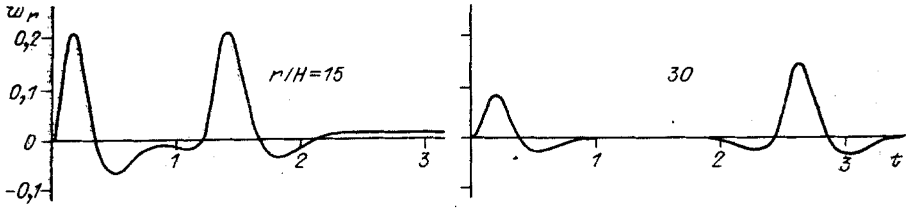

Fig. 1

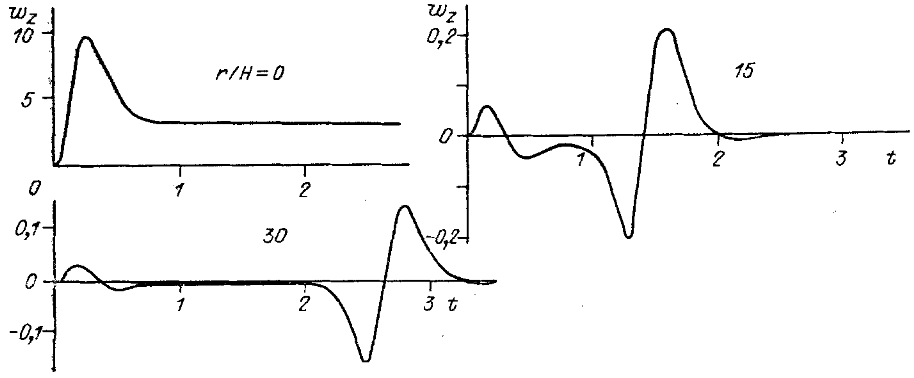

Fig. 2

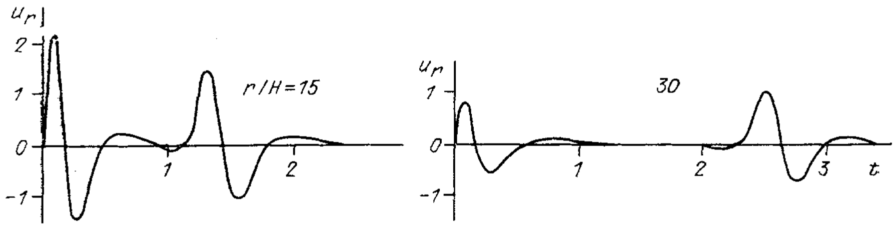

Fig. 3

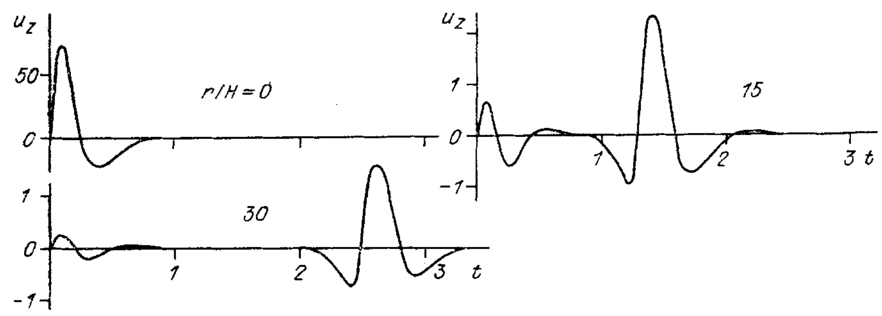

Fig. 4

Here $f_2(t)$ is the source function, which is proportional to the second time derivative of the reduced potential:

$$f_2(t) = \ddot{\Phi}_0(t)/R_0 c_p. \tag{4.2}$$

The curves shown in Figs. 1-4 have the usual form described in [1]. They are typified by the presence of spikes at $r/H \gtrsim 5$. The first spike, which decays comparatively rapidly with increasing distance from the detonation center, is created by the arrival of the seismic wave front at the observation point; the second spike, which decays more slowly, corresponds to a Rayleigh surface wave. The time time interval between the two spikes is approximately proportional to the epicentral distance; the width of the Rayleigh spike depends weakly on the distance in the case of the short-lived sources considered here.

This situation is attributable to the fact that the profile of the Rayleigh wave for such sources is determined mainly by the fundamental solution. Indeed, if the active time of the source is short in comparison with the characteristic time scale of the fundamental solution and if, on the other hand, the arrival time of the Rayleigh wave at the observation point, reckoned from the arrival of the wave front, is large, then the functions $G_{r,z}$ in Eq. (4.1) can be expanded in a power series in τ, and infinity can be taken as the upper limit of integration. Taking Eq. (4.2) into account and retaining only the principal term in the required estimate, we obtain $w_{r,z}(t) \approx [\Phi_0(\infty)/R_0C_p]G_{r,z}(t)$. This asymptotic estimate illustrates the foregoing considerations and shows that the amplitude of the Rayleigh wave far from the detonation center in the case of short-lived sources is proportional to $\Phi_0(\infty)$, i.e., is proportional to the power of the source.

These assertions become meaningless for long-lasting sources of the earthquake type. Experiments [9] show that the period of the surface waves grows in this case. This is also readily confirmed analytically by setting $\Omega_H \sim 1\ \text{sec}^{-1}$.

In closing, we regret having to say that our calculations do not agree with those in [8], where an attempt is made to obtain expressions for the Green's function that will lead to fulfillment of the causality principle. We note that our proposed approach meets this requirement, because the fundamental solution (2.1) is in fact nonvanishing only after the wave front, i.e., $R_f \geqslant R$. A far more complicated expression is obtained in [8] for the solution and is difficult to analyze either analytically or numerically. Consequently, the reason for these discrepancies has eluded us. It could be both the lack of equivalence between the expressions derived here and in [8] for the Green's functions and also insufficient accuracy in the numerical calculations.

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